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A THEOREM ON (m, n) CORRESPONDENCES.

By L. I. NEIKIRK.

Emil Weyr in a paper* published in 1870 defines an n fold involution of points on a line as the points whose abscissas are roots of the equations $f(x) - \lambda\varphi(x) = 0$, where $f(x)$ and $\varphi(x)$ are rational integral functions of x, and λ is a variable parameter. He finds its double elements and other properties.

In a second paper† he introduces two involutions $f(x) - \lambda\varphi(x) = 0$, and $F(y) - \kappa\psi(y) = 0$ of degrees m and n, and terms these involutions projective when $A\lambda\kappa + B\lambda + C\kappa + D = 0$. The two involutions group the points of the two lines into sets of m and n points each, and the last equation puts these sets into a (1, 1) correspondence.

He considers the x and y as Cartesian coördinates of a point and develops the properties of the curves defined. Among other properties is the following; let the line $x = x_1$ cut the curve in the points $(x_1 : y_j)$ $j = 1, 2, \cdots n$ and the line $y = y_1$ cut the curve in the points $(x_i : y_1)$ $i = 1, 2, \cdots m$, then the curve will pass through all points of the lattice work

$$(A) \qquad x = x_i, \quad y = y_j, \quad i = 1, 2, \cdots m, \quad j = 1, 2, \cdots n.$$

The purpose of this paper is to prove the following theorem.

Theorem. *If an (m, n) correspondence is given by a rational integral equation $f(x, y) = 0$ of degree m in x and n in y, and has a single correspondence of the type $(x_i : y_j)$, $i = 1, 2, \cdots m$, $j = 1, 2, \cdots n$, then $f(x, y) = 0$ can be reduced to the form*

$$\varphi(x) + \lambda\theta(x) = 0, \quad \psi(y) + \lambda\omega(y) = 0,$$

where φ, θ, ψ, and ω are rational integral functions, the first two of degree m and the last two of degree n, and λ is a variable parameter, and every correspondence is of the type $(x_i : y_j)$.

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[*] Sitzungsberichte der k. Böhmischen Gesellschaft der Wissenschaften in Prag. (1870), 14–19.

[†] Mathematische Annalen, III (1870), 34–44.

This theorem shows that if a rational algebraic curve $f(x, y) = 0$ passes through all points of *one such lattice work as* (A) it will pass through all points of *every such lattice work.*

Proof. Let $f(x, y) = A_0(x)y^n + A_1(x)y^{n-1} + \cdots + A_n(x)$ where the A's are rational functions of x of degree m or less.

Let

$$f(x_1, y) = a_0 y^n + a_1 y^{n-1} + \cdots + a_n,$$

then since

$$f(x_i, y_j) = 0, \quad i = 1, 2, \cdots m, \quad j = 1, 2, \cdots n.$$

it follows that

$$f(x_i, y) = k_i f(x_1, y),$$

and

$$A_s(x_i) = k_i a_s, \quad i = 1, 2, \cdots m, \quad s = 1, 2, \cdots n,$$

k_1 being equal to 1.

The expressions $A_s(x)/a_s$ take the m values $k_1 = 1, k_2, \cdots k_m$, when x takes the values $x_1, x_2, \cdots x_m$. The exceptional case $a_s = 0$ will be treated later.

In Lagrange's interpolation formula let

$$\varphi(x) = (x - x_1)(x - x_2)\cdots(x - x_m) \qquad [x_r \neq x_s, r \neq s],$$

$$\theta(x) = \sum_{r=1}^{r=m} \frac{k_r \varphi(x)}{(x - x_r)\varphi'(x_r)}.$$

Then $\theta(x_i) = k_i$ and the most general rational integral function of degree m which takes the m values $k_1, k_2, \cdots k_m$ when x takes the values $x_1, x_2, \cdots x_m$ is $\theta(x) + l\varphi(x)$.

Therefore

$$\frac{A_s(x)}{a_s} = \theta(x) + l_s\varphi(x), \qquad s = 1, 2, \cdots, n,$$

and

$$f(x, y) = (a_0 y^n + a_1 y^{n-1} + \cdots + a_n)\theta(x) + (a_0 l_0 y^n + a_1 l_1 y^{n-1} + \cdots + a_n l_n)\varphi(x)$$

$$= \psi(y)\theta(x) - \omega(y)\varphi(x)$$

If $a_s = 0$, then

$$A_s(x_i) = 0, \qquad i = 1, 2, \cdots, m,$$

so that

$$A_s(x) = l_s\varphi(x).$$

In every case therefore the correspondence is given by an equation of the form

$$\psi(y)\theta(x) - \omega(y)\varphi(x) = 0.$$

From this we get

$$\frac{\varphi(x)}{\theta(x)} = \frac{\psi(y)}{\omega(y)} = -\lambda,$$

and the theorem follows

$$\varphi(x) + \lambda\theta(x) = 0, \quad \psi(y) + \lambda\omega(y) = 0.$$

URBANA, ILLINOIS.